

# DIVISION ALGORITHMS

Edward Cannon 2008

The following are five algorithms for division that have been used throughout the ages. These algorithms are not all the algorithms used, but are ones I have had experience with. Each algorithm has its pros and cons, and each one may be easier for one person and harder for another.

## Classic Long Division

This is long division as taught to most students in the United States. Strengths of this algorithm are compactness on the page and speed. Disadvantages are that to be efficient, a strong knowledge of multiplication facts is required.

*Example 468 divided by 4*

$$\begin{array}{r} 117 \\ 4 \overline{) 468} \\ \underline{- 4} \phantom{0} \\ 06 \\ \underline{- 4} \phantom{0} \\ 28 \\ \underline{- 28} \\ 0 \end{array}$$

The largest multiple of 4 less than or equal to 4 is 4

The largest multiple of 4 less than or equal to 6 is 4

The largest multiple of 4 less than or equal to 28 is 28

## Modified Long Division, or Table Division

This is a slight modification of classic long division that requires less knowledge of multiplication facts and may be simpler for some students to grasp. This method is most practical when the divisor is large, and its multiples are not already known.

*Example 468 divided by 4*

$$\begin{array}{r} 117 \\ 4 \quad 4 \overline{) 468} \\ 8 \quad - 4 \\ \underline{12} \quad 06 \\ 16 \quad - 4 \\ \underline{20} \quad 8 \\ 24 \quad - 28 \\ \underline{28} \quad 0 \end{array}$$

The largest multiple of 4 less than or equal to 4 is 4

The largest multiple of 4 less than or equal to 6 is 4

The largest multiple of 4 less than or equal to 28 is 28

Multiplication table constructed by repeated addition

## Turret Division

Turret division is an old division algorithm that was taught in the 15<sup>th</sup> century at schools for Italian merchant's sons.

*Example 468 divided by 4*

	1	
	2	
	1 1 4	Columns in answer are added to arrive at the final answer, 117
4	4 6 8	
-	4	4 goes into 4
	0 6	Nothing left over, bring down 6
-	4	4 goes into 6
	2 8	4 does not go into 2, bring down 8
-	1 6	4 times 4 is 16, less than 28
	1 2	4 goes into 12 at least 2 times
	8	
	4	4 goes into 4 1 time
	4	
	0	

## Partition Division

fill

## Prime Factorization Division

The essence of prime factorization division is to construct the prime factorizations of both the divisor and the dividend and then to reduce the problem to a simpler problem involving memorized division facts and simple multiplication.

*Example: 468 divided by 18*

The prime factorization of 468 is  $2 \cdot 2 \cdot 3 \cdot 3 \cdot 13$  or  $2^2 \cdot 3^2 \cdot 13$ . The prime factorization of 18 is  $2 \cdot 3^2$ . Both factorizations include  $2 \cdot 3^2$  so these terms can be removed leaving  $2 \cdot 13 \div 1$  or 26.

*Example: 5600 divided by 140*

The prime factorization of 5600 is  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 = 2^5 \cdot 5^2 \cdot 7$ . The prime factorization of 140 is  $2 \cdot 2 \cdot 5 \cdot 7 = 2^2 \cdot 5 \cdot 7$ . Both factorizations include  $2^2 \cdot 5 \cdot 7$  so these terms can be removed leaving the simplified problem  $2^3 \cdot 5 \div 1 = 40$ .

The real difficulty is in constructing the prime factorizations and the risk that reduction will be impossible. If the divisor and dividend are coprime, this method is useless.

## Fraction Division

Fraction division is a variation of Prime Factorization Division that writes the division problem as a fraction and then attempt to simplify the problem. The advantages over prime factorization are that all prime factors do not need to be known.

*Example: 144 divided by 60*

Write the problem as a fraction  $\frac{144}{60}$  then try and reduce the fraction. 144 and 60 are both even, so by multiple reductions we get  $\frac{72}{30} = \frac{36}{15} = \frac{12}{5}$  which is ten either written as a mixed number  $2\frac{2}{5}$  or divided by long division to generate a decimal, 2.4.